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Generating solutions of a linear equation and structure of elements of the Zelisko group

V.A. Bovdi, V.P. Shchedryk

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Abstract

Solutions of a linear equation $b=ax$ in a homomorphic image of a commutative Bézout domain of stable range 1.5 are developed. It is proved that the set of solutions of a solvable linear equation contains at least one solution that divides the rest, which is called a generating solution. Generating solutions are pairwise associates. Using this result, the structure of elements of the Zelisko group is investigated.

Keywords

Linear equation; Commutative Bézout domain; Stable range; Zelisko group

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