
LINEAR ALGEBRA AND ITS APPLICATIONS

Founded by

A. J. Hoffman, A. S. Householder, A. M. Ostrowski,
H. Schneider, and O. Taussky Todd

Editors-in-Chief

Alan J. Hoffman: 1968–1972

Hans Schneider: 1972–1978

Richard A. Brualdi and Hans Schneider: 1979–1999

Richard A. Brualdi, Volker Mehrmann, and Hans Schneider: 1999–2010

Richard A. Brualdi, Volker Mehrmann, Hans Schneider, and Peter Šemrl:
2011–2012

Richard A. Brualdi, Volker Mehrmann, and Peter Šemrl: 2013–

Editors-in-Chief

Richard A. Brualdi

Department of Mathematics
The University of Wisconsin
Madison, Wisconsin 53706
brualdi@math.wisc.edu

Volker Mehrmann

Inst. für Mathematik, MA 4-5
Strasse des 17. Juni 136
D-10623 Berlin, FRG
mehrmann@math.tu-berlin.de

Peter Šemrl

Department of Mathematics
University of Ljubljana
Jadranska 19
SI-1000 Ljubljana, Slovenia
peter.semrl@fmf.uni-lj.si

Associate Editors

Sebastian M. Cioaba

University of Delaware
Department of Mathematical Sciences
Ewing Hall
15 Orchard Road
Newark, DE 19716-2553
cioaba@udel.edu

Andreas Frommer

Fachbereich Mathematik und
Naturwissenschaften
Bergische Universität Wuppertal
42097 Wuppertal, Germany
frommer@math.uni-wuppertal.de

J.M. Landsberg

Department of Mathematics
Texas A&M University
College Station, TX 77845-3368
jml@math.tamu.edu

Tobias Damm

Department of Mathematics,
TU Kaiserslautern D-67663
Kaiserslautern, Germany
damm@mathematik.uni-kl.de

Heide Gluesing-Luerssen

Dept. of Mathematics
University of Kentucky
715 Patterson Office Tower
Lexington, KY 40506, USA
heide.gl@uky.edu

Lek-Heng Lim

Department of Statistics
University of Chicago
Chicago, IL 60637, USA
lekheng@uchicago.edu

Shaun Fallat

Department of Mathematics
and Statistics
University of Regina
Regina, Saskatchewan
Canada S4S 0A2
sfallat@math.uregina.ca

Leslie Hogben

Department of Mathematics
Iowa State University
Ames, IA 50011
hogben@aimath.org

Christian Mehl

TU Berlin
Institut für Mathematik
Sekretariat MA 4-5
10623 Berlin, Germany
mehl@mail.math.tu-berlin.de

Volume 596, 1 July 2020



ELSEVIER

Amsterdam – Boston – London – New York – Oxford – Paris – Philadelphia – San Diego – St. Louis

Beatrice Meini
Dipartimento di Matematica "Tonelli"
Largo Bruno Pontecorvo, 5
56127 Pisa, Italy
meini@dm.unipi.it

Vladimir Muller (Müller)
Institute of Mathematics AS CR
Žitná 25
CZ - 115 67 Praha 1
Czech Republic
muller@math.cas.cz

Deanna Needell
Department of Mathematics
University of California, Los Angeles
Los Angeles, CA 90095, USA
deanna@math.ucla.edu

João Filipe Queiroz
Departamento de Matemática
Universidade de Coimbra
Apartado 3008
3001-454 Coimbra, Portugal
jfqueiro@mat.uc.pt

Holger Rauhut
Lehrstuhl C für Mathematik (Analysis)
RWTH Aachen University
Templergraben 55
52062 Aachen, Germany
rauhut@mathc.rwth-aachen.de

Bryan L. Shader
Department of Mathematics
University of Wyoming
Laramie, WY 82071
bshader@uwyo.edu

Senior Editors

Harm Bart
Erasmus University
Econometric Institute
P.O. Box 1738, 3000 DR Rotterdam
The Netherlands
bart@ese.eur.nl

Michele Benzi
Classe di Scienze
Scuola Normale Superiore
Piazza dei Cavalieri, 7
56126 Pisa, Italy
michele.benzi@sns.it

Albrecht Böttcher
Fakultät für Mathematik
TU Chemnitz
D-09107 Chemnitz, Germany
aboettch@mathematik.tu-chemnitz

Matej Brešar
Department of Mathematics
University of Ljubljana
Jadranska 19, 1000 Ljubljana
Slovenia
matej.bresar@fmf.uni-lj.si

Froilán M. Dopico
Department of Mathematics
Universidad Carlos III de Madrid
Avenida de la Universidad
30 28911 Leganés, Madrid, Spain
dopico@math.uc3m.es

Distinguished Editors

T. Ando, Sapporo, Japan
Rajendra Bhatia, New Delhi, India
Chandler Davis, Toronto, Canada
Ludwig Elsner, Bielefeld, Germany
Paul A. Fuhrmann, Beer sheva, Israel

Jia-yu Shao
Department of Mathematics
Tongji University
Shanghai 200092, China
jyshao@tongji.edu.cn

Dragan Stevanović
Mathematical Institute
Serbian Academy of Sciences and Arts
Kneza Mihaila 36
11001 Belgrade
Serbia
dragance106@yahoo.com

Zdeněk Strakoš
Institute of Computer Science
Academy of Sciences of the Czech Republic
Pod Vod. věží 2, 182 07 Prague 8
Czech Republic
strakos@cs.cas.cz

Françoise Tisseur
Dept. of Mathematics
University of Manchester
Oxford Road, M13 9PL
Manchester, England, UK
francoise.tisseur@manchester.ac.uk

Christiane Tretter
Math. Institute
University of Bern
Sidlerstr. 5, 3012 Bern, Switzerland
tretter@math.unibe.ch

Michael Tsatsomeros
Department of Mathematics
Washington State University
Pullman, WA 99164
tsat@math.wsu.edu

Heike Fassbender
AG Numerik
Institut Computational Mathematics
Carl-Friedrich-Gauss-Fakultät für
Mathematik und Informatik
TU Braunschweig
D-38023 Braunschweig, Germany
h.fassbender@tu-bs.de

Shmuel Friedland
Department of Mathematics,
Statistics & Computer Science
University of Illinois at Chicago
851 S. Morgan St.
Chicago, Illinois 60607-7045
friedlan@uic.edu

Robert Guralnick
Department of Mathematics
University of Southern California
Los Angeles, CA 90089-2532
guralnic@math.usc.edu

N.J. Higham
School of Mathematics
University of Manchester
Manchester, M13 9PL, UK
higham@ma.man.ac.uk

Stephen Kirkland
Department of Mathematics
University of Manitoba
Winnipeg, MB, R3T 2N2, Canada
Stephen.Kirkland@umanitoba.ca

Alan J. Hoffman, Yorktown Heights,
New York
Roger A. Horn, Tampa, Florida
Thomas Kailath, Stanford, California
Thomas J. Laffey, Dublin, Ireland

Eugene Tyrtyshnikov
Inst. of Numerical Mathematics
Russian Academy of Sciences
Ul. Gubkina 8
Moscow 117333, Russia
eugene.tyrtyshnikov@gmail.com

André Uschmajew
Max Planck Institute for Mathematics in the
Sciences
04103 Leipzig, Germany
uschmajew@mis.mpg.de

Bart Vandereycken
Dept. of Mathematics
University of Geneva
Case postale 64
1211 Genève 4, Switzerland
bart.vandereycken@unige.ch

Xingzhi Zhan
Department of Mathematics
East China Normal University
Shanghai 200241, China
zhan@math.ecnu.edu.cn

Chi-Kwong Li
Department of Mathematics
College of William and Mary
P.O. Box 8795
Williamsburg, VA 23187-8795
ckli@math.wm.edu

Raphael Loewy
Department of Mathematics
Technion-Israel Institute of Technology
Haifa 32000, Israel
loewy@technion.technion.ac.il

Daniel B. Szyld
Department of Mathematics (038-16)
Temple University
Philadelphia, PA 19122, USA
szyld@temple.edu

Vladimir V. Sergeichuk
Institute of Mathematics
Tereshchenkivska 3
01601, Kiev-4, Ukraine
sergeich@imath.kiev.ua

Peter Lancaster, Calgary, Canada
Beresford N. Parlett, Berkeley, California
G.W. Stewart, College Park, Maryland
Richard S. Varga, Kent, Ohio

Contents

On triangular similarity of nilpotent triangular matrices

Ming-Cheng Tsai, Meaza Bogale, Huajun Huang
Pages 1-35

A reduction procedure for the Colin de Verdière number of a graph

Lon Mitchell, Irene Sciriha
Pages 36-48

m -Isometric operators and their local properties

Zenon Jan Jabłoński, Il Bong Jung, Jan Stochel
Pages 49-70

On the ABC spectra radius of unicyclic graphs

Xueliang Li, Junming Wang
Pages 71-81

Operators on positive semidefinite inner product spaces

Victor A. Bovdi, Tetiana Klymchuk, Tetiana Rybalkina, Mohamed A. Salim,
Vladimir V. Sergeichuk
Pages 82-105

The space of all p -th roots of a nilpotent complex matrix is path-connected

Clément de Seguins Pazzis
Pages 106-116

The Terwilliger algebra of the Grassmann scheme $J_q(N, D)$ revisited from the viewpoint of the quantum affine algebra $U_q(\mathfrak{sl}^2)$

Xiaoye Liang, Tatsuro Ito, Yuta Watanabe
Pages 117-144

On the sparseness of certain linear MRD codes

Heide Gluesing-Luerssen
Pages 145-168

On some open questions concerning determinantal inequalities

Mohammad M. Ghabries, Hassane Abbas, Bassam Mourad
Pages 169-183

Adjacency spectra of random and complete hypergraphs

Joshua Cooper
Pages 184-202

Positivity properties of some special matrices

Priyanka Grover, Veer Singh Panwar, A. Satyanarayana Reddy
Pages 203-215

A linear code based on resilient Boolean maps whose dual is a platform for a robust secret sharing scheme

Juan Carlos Ku-Cauich, Guillermo Morales-Luna
Pages 216-229

A local-global principle for linear dependence in enveloping algebras of Lie algebras

Jaka Cimprič, Aljaž Zalar
Pages 230-255

'Hubs-repelling' Laplacian and related diffusion on graphs/networks

Ernesto Estrada
Pages 256-280

ISSN: 0024-3795

Copyright © 2021 Elsevier Inc. All rights reserved

Operators on positive semidefinite inner product spaces

Victor A. Bovdi, Tetiana Klymchuk, Tetiana Rybalkina, Mohamed A. Salim,
Vladimir V. Sergeichuk

<https://doi.org/10.1016/j.laa.2020.03.004>

Abstract

Let U be a semiunitary space; ie, a complex vector space with scalar product given by a positive semidefinite Hermitian form $\langle \cdot, \cdot \rangle$. If a linear operator $A: U \rightarrow U$ is bounded (ie, $\|A u\| \leq c \|u\|$ for some $c \in \mathbb{R}$ and all $u \in U$), then the subspace $U_0 := \{u \in U \mid \langle u, u \rangle = 0\}$ is invariant, and so A defines the linear operators $A_0: U_0 \rightarrow U_0$ and $A_1: U/U_0 \rightarrow U/U_0$.

Let A be an indecomposable bounded operator on U such that $0 \neq U_0 \neq U$. Let λ be an eigenvalue of A_0 . We prove that the algebraic multiplicity of λ in A_1 is not less than the geometric multiplicity of λ in A_0 , and the geometric multiplicity of λ in A_1 is not less than the number of Jordan blocks $J_t(\lambda)$ of each fixed size $t \times t$ in the Jordan canonical form of A_0 .

We give canonical forms of selfadjoint and isometric operators on U , and of Hermitian forms on U .

For an arbitrary system of semiunitary spaces and linear mappings on/between them, we give an algorithm that reduces their matrices to canonical form. Its special cases are Belitskii's and Littlewood's algorithms for systems of linear operators on vector spaces and unitary spaces, respectively.

Keywords

Positive semidefinite inner product spaces; Bounded operators; Selfadjoint and isometric operators; Belitskii's and Littlewood's algorithms

References

- [1] T.Ya. Azizov, I.S. Iokhvidov, *Linear Operators in Spaces with an Indefinite Metric*, John Wiley & Sons, Ltd., Chichester, 1989.
- [2] G.R. Belitskiĭ, Normal forms in a space of matrices, in: V.A. Marchenko (Ed.), *Analysis in Infinite Dimensional Spaces and Operator Theory*, Naukova Dumka, Kiev, 1983, pp. 3–15 (in Russian).
- [3] G.R. Belitskii, Normal forms in matrix spaces, *Integral Equations Operator Theory* 38 (2000) 251–283.
- [4] G.R. Belitskii, V.V. Sergeichuk, Complexity of matrix problems, *Linear Algebra Appl.* 361 (2003) 203–222.
- [5] R. Benedetti, P. Cragolini, Versal families of matrices with respect to unitary conjugation, *Adv. in Math.* 54 (1984) 314–335.
- [6] J. Bognar, *Indefinite Inner Product Spaces*, Springer-Verlag, New York/Heidelberg, 1974.
- [7] T. Bruestle, V.V. Sergeichuk, Estimate of the number of one-parameter families of modules over a tame algebra, *Linear Algebra Appl.* 365 (2003) 115–133.
- [8] Y. Chen, L. Nie, Y. Xu, Belitskii’s canonical forms of linear dynamical systems, *Linear Algebra Appl.* 531 (2017) 533–536.
- [9] Y. Chen, Y. Xu, H. Li, W. Fu, Belitskii’s canonical forms of upper triangular nilpotent matrices under upper triangular similarity, *Linear Algebra Appl.* 506 (2016) 139–153.
- [10] P. Gabriel, L.A. Nazarova, A.V. Roiter, V.V. Sergeichuk, D. Vossieck, Tame and wild subspace problems, *Ukrainian Math. J.* 45 (1993) 335–372.
- [11] F.R. Gantmacher, *The Theory of Matrices*, Vol. 2, AMS Chelsea Publishing, Providence, RI, 2000.
- [12] I. Gohberg, P. Lancaster, L. Rodman, *Indefinite Linear Algebra and Applications*, Birkhäuser, Boston, 2006.
- [13] R.A. Horn, C.R. Johnson, *Matrix Analysis*, 2nd ed., Cambridge University Press, Cambridge, 2013.
- [14] D.E. Littlewood, On unitary equivalence, *J. London Math. Soc.* 28 (1953) 314–322.
- [15] C. Mehl, A.C.M. Ran, L. Rodman, Hyponormal matrices and semidefinite invariant subspaces in indefinite inner products, *Electron. J. Linear Algebra* 11 (2004) 192–204.
- [16] C. Mehl, A.C.M. Ran, L. Rodman, Semidefinite invariant subspaces: degenerate inner products, *Operator Theory: Advances and Appl.*, Current trends in operator theory and its applications, 149 (2004) 467–486.
- [17] C. Mehl, L. Rodman, Symmetric matrices with respect to sesquilinear forms, *Linear Algebra Appl.* 349 (2002) 55–75.
- [18] V.V. Sergeichuk, Classification of linear operators in a finite dimensional unitary space, *Funct. Anal. Appl.* 18 (3) (1984) 224–230.
- [19] V.V. Sergeichuk, Classification problems for systems of forms and linear mappings, *Math. USSR-Izv.* 31 (no. 3) (1988) 481–501. Theorem 2 over quaternions was corrected in arxiv.org/abs/0801.0823.

- [20] V.V. Sergeichuk, Unitary and Euclidean representations of a quiver, *Linear Algebra Appl.* 278 (1998) 37–62.
- [21] V.V. Sergeichuk, Canonical matrices for linear matrix problems, *Linear Algebra Appl.* 317 (2000) 53–102.
- [22] V.V. Sergeichuk, D.V. Galinskiĭ, Classification of pairs of linear operators in a four-dimensional vector space, *Infinite groups and related algebraic structures*, Akad. Nauk Ukrainy, Inst. Mat., Kiev, 1993, 413–430 (in Russian).
- [23] H. Shapiro, A survey of canonical forms and invariants for unitary similarity, *Linear Algebra Appl.* 147 (1991) 101–167.